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Abstract

Using a standard linear city model with two firms, we consider how their licensing activities following their R&D investments affect the locations of the firms and the effort levels of the R&D investments. Although recent studies show that R&D investments that may cause a large cost differential between firms yield agglomeration of the firms, we show that licensing activities after R&D investments always lead to the maximum differentiation between the firms and mitigate the price competition. We also show that those licensing activities induce the socially optimal effort levels of R&D activities.

JEL classification: L13, O32, R32

Key words: licensing, oligopoly, R&D, location

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1 Introduction

Since the seminal work of Hotelling (1929), the model of spatial competition, which is one of the most important models of oligopoly, has been seen by many subsequent researchers as an attractive framework for analyzing product differentiation.[1] The major advantage of this approach is that it allows an explicit analysis of product selection. Of particular interest is the equilibrium pattern of product locations and the degree of product differentiation. The original finding of Hotelling (1929) is that firms produce similar products (minimum differentiation). d'Aspremont *et al.* (1979) consider two-stage location–price games. They show that products are maximally differentiated when transport costs are quadratic. The firms in their study never choose minimal differentiation, to avoid cutthroat competition in the subsequent stage.

Recently, several researchers have introduced quality uncertainty into the duopoly location model. This uncertainty stems from R&D activities. Gerlach *et al.* (2005) and Christou and Vettas (2005) provide interesting results concerning the relation between technological fluctuation (the uncertainty of the outcomes of R&D activities) and product positioning. They mainly consider the following three-stage game: first, each firm chooses its location in a linear city; second, each firm's product quality is probabilistically determined; and third, each firm sets its price. They show it is possible that spatial agglomeration (minimum differentiation) appears in equilibrium while the basic structures of their models differ from each other. In Gerlach *et al.* (2005), if the product quality of a firm is zero (the project is unsuccessful), the firm remains inactive in the market. In Christou and Vettas (2005), each firm has the ability to produce its product. Depending on the quality difference determined probabilistically and the locations of the firms, the quantities supplied by the firms are determined.[2]

[1] See, d'Aspremont *et al.* (1979), Neven (1985), and Anderson *et al.* (1992), among others.

[2] Some studies have already shown that minimum differentiation may appear in equilibrium. Price collusion after firms have made location choices is considered in Friedman and Thisse (1993). Cooperation between firms is considered in the form of information exchange through communication by Mai and Peng (1999). de Palma *et al.* (1985) introduce unobservable attributes in brand choice and consider situations in which central agglomeration does not induce the standard Bertrand competition with homogeneous products. They show that sufficient heterogeneity between firms induces central agglomeration. Bester (1998) introduces

We suppose that a firm undertakes R&D to develop an advanced technology to produce its product. We often observe that such a firm transfers its technology to its rivals, subject to a fee. That is, after the outcome of R&D is determined, firms having advanced technologies often earn additional profits from licensing fees. Neither Gerlach *et al.* (2005) nor Christou and Vettas (2005) discuss these licensing activities, which are potentially important from the viewpoint of product positioning.

One purpose of our paper is to consider how licensing activities affect the locations of firms. That is, we introduce the following stage into the models of Gerlach *et al.* (2005) and Christou and Vettas (2005): a firm that has a cost advantage over its rival might license its technology after their model's second stage.

We consider how licensing activities following R&D affect the product choices of firms. There exist several modes of licensing: a royalty per unit of output produced with the transferred technology, a fixed fee that is independent of the quantity supplied with transferred technology, or a hybrid such as a two-part tariff. In our paper, we consider a royalty per unit of output produced with the transferred technology.[3] Contrary to the results in Gerlach *et al.* (2005) and Christou and Vettas (2005), we show that the maximum differentiation always appears in equilibrium when each firm is able to transfer its own advanced technology to its rival via licensing.

Since the seminal work of Arrow (1962), there is a vast literature (see Kamien (1992)), which focuses on the licensing arrangement optimal for the patentee in a wide variety of situations. There are papers that analyze various aspects of patent licensing (for instance, insider or outsider patentees, incentives for innovation, and market structures (monopoly, oligopoly, and competitive)). Although most of the papers use a standard framework of price and quantity competition, endogenous product positioning is not considered.[4]

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vertical quality characteristics, asymmetric information about this quality between sellers and consumers, and a limited number of repeat purchases by consumers. Bester (1998) finds that, in equilibrium, central agglomeration appears if the number of repeat purchases is more than one and not too large. Gerlach *et al.* (2005) and Christou and Vettas (2005) also deal with models of endogenous production costs (quality) with spatial competition. See, for instance, Brekke and Straume (2004) and Matsushima (2004, 2006).

[3] Two-part tariff licensing is chosen by firms in our model.

[4] In the literature of licensing, the interactions between the patentee and licensee(s) are usually discussed

To our knowledge, Poddar and Sinha (2004) are alone in employing the Hotelling model with cost asymmetry in the analysis of licensing.[5] Although they consider the optimal licensing strategy of an outsider patentee as well as an insider, the locations of the firms are exogenously given. We think, therefore, that the setting discussed here provides additional insight for the licensing literature. In addition, there is another important difference between this paper and Poddar and Sinha (2004). In our paper, the firm that holds the patent chooses its price to maximize total profits (profits from its own products plus revenue from license fee). On the other hand, Poddar and Sinha (2004) assume that the firm chooses its price so as to maximize its profit from its own products and neglects license fee revenue (although the firm maximizes total profit when it chooses the level of the license fee).[6] We show that one of their results on nonneutrality does not hold in a more plausible setting.

We also consider whether the level of R&D activity under such a licensing scheme is excessive or insufficient from the viewpoint of social welfare. We show that the incentive for R&D investment by the firm is consistent with the additional social benefit of R&D investment. That is, efficient R&D investments is achieved by firms without any government intervention.

The remainder of this paper is organized as follows. Section 2 presents the basic model. Section 3 shows the result under the exogenous location model. Section 4 shows the result under the endogenous location model. Section 5 shows the main result of the paper. Sections 6 and 7 discuss the levels of R&D investments, and Section 8 concludes.

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and whether the licensee(s) are insider(s) or outsider(s) is important (see, for instance, Hernández-Murilloa and Llobet (2006), Kamien et al. (1992), Kamien and Tauman (1984, 2002), Katz and Shapiro (1986), Muto (1993), Sen and Tauman (2007), Wang (1998, 2002), and Wang and Yang (1999), among others).

[5] Some papers on licensing include the role of product differentiation. See, for instance, Caballero-Sanz *et al.* (2002), Faulí-Oller and Sandonís (2002), Mukherjee and Balasubramanian (2001), Muto (1993), Wang and Yang (1999). Those papers do not use location models except for Caballero-Sanz *et al.* (2002) which treat a location model with exogenous location choices.

[6] Poddar and Sinha (2004) do not explicitly state this assumption, but their result does not hold without this assumption. See also footnote 9 of our paper.

2 The model

Consider a linear city along the unit interval $[0,1]$, where Firm 1 is located at x_1 and Firm 2 is located at $1-x_2$. Without loss of generality, we assume that $x_1 \leq 1-x_2$. Consumers are uniformly distributed along the interval. Each consumer buys exactly one unit of the good, which can be produced by Firms 1 and 2. The utility of the consumer located at x is given by:

$$u_x = \begin{cases} -T(|x_1 - x|) - p_1 & \text{if bought from Firm 1,} \\ -T(|1 - x_2 - x|) - p_2 & \text{if bought from Firm 2,} \end{cases} \tag{1}$$

where $T(\cdot)$ represents the transport cost incurred by the consumer. $T(\cdot)$ is an increasing function. For a consumer living at $x(p_1, p_2, x_1, x_2)$, where

$$-T(|x_1 - x(p_1, p_2, x_1, x_2)|) - p_1 = -T(|1 - x_2 - x(p_1, p_2, x_1, x_2)|) - p_2, \tag{2}$$

total utility is the same whichever of the two firms is chosen. Thus, the demand facing Firm 1, D_1, and that facing Firm 2, D_2, are given by:

$$\begin{array}{l} D_1(p_1, p_2, x_1, x_2) = \min\{\max(x(p_1, p_2, x_1, x_2), 0), 1\}, \\ D_2(p_1, p_2, x_1, x_2) = 1 - D_1(p_1, p_2, x_1, x_2). \end{array} \tag{3}$$

We now assume that one of the firms (denoted as Firm 1) has a cost-reducing innovation. Assume preinnovation marginal costs of both firms are $c_1 = c_2 = c$. Firm 1's cost-reducing innovation lowers its marginal cost by $d > 0$, so that postinnovation, $c_1 = c - d$ and $c_2 = c$.

Firm 1 now licenses its new technology to Firm 2 at a royalty rate (r). The total royalty Firm 2 pays will depend on the quantity supplied by Firm 2 using the new technology. In this case, Firm 1's marginal cost of production is $c - d$ and Firm 2's marginal cost of production becomes $c - d + r$. The royalty $r \leq d$, otherwise Firm 2 will not accept the contract.

The game runs as follows. In the first stage, both firms i choose their location $x_i \in [0,1]$ simultaneously. In the second stage, r is determined (Firm 1 offers the royalty r for Firm 2. Firm 2 chooses whether or not to accept it).[7] In the third stage, the firms i choose their prices $p_i \in [c_i, \infty)$ simultaneously.[8]

[7] The order of the first and second stages is interchangeable . If Firm 1 chooses r and then the firms choose their locations, all our Section 3 and 4 results hold true.

[8] Choosing $p_i < c_i$ is weakly dominated by choosing $p_i = c_i$, so we assume that the lower bound of the price is its cost.

3 Result: Exogenous locations

To clarify the effect of royalty licensing on the quantities supplied by the firms, we first discuss the case in which their locations are exogenously given by $x_1 = x_2 = 0$. That is, we assume that the firms locate at the edges respectively. We now label $x(p_1, p_2, x_1, x_2)$ $x(p_1, p_2)$. The profit functions of the firms are:

$$\begin{aligned}
\pi_1 &= (p_1 - (c-d))x(p_1, p_2) + r(1 - x(p_1, p_2)) \\
&= (p_1 - (c-d) - r)x(p_1, p_2) + r, \\
\pi_2 &= (p_2 - (c-d) - r)(1 - x(p_1, p_2)).
\end{aligned}$$

The first-order conditions are:

$$\begin{aligned}
\frac{\partial \pi_1}{\partial p_1} &= x(p_1, p_2) + (p_1 - (c-d) - r)\frac{\partial x(p_1, p_2)}{\partial p_1} \\
&= x(p_1, p_2) - (p_1 - (c-d) - r) \times \frac{1}{T'(1 - x(p_1, p_2)) + T'(x(p_1, p_2))}, \\
\frac{\partial \pi_2}{\partial p_2} &= 1 - x(p_1, p_2) - (p_2 - (c-d) - r) \times \frac{1}{T'(1 - x(p_1, p_2)) + T'(x(p_1, p_2))}.
\end{aligned}$$

The first-order conditions lead to:

$$p_1 = p_2 = c - d + r + T'\left(\frac{1}{2}\right), \quad x(p_1, p_2) = \frac{1}{2}.$$

The profits of the firms are:

$$\pi_1 = \frac{T'(1/2)}{2} + r, \quad \pi_2 = \frac{T'(1/2)}{2}. \tag{4}$$

We find that the quantities supplied by the firms do not depend on the level of the royalty fee, r. We also find that Firm 2's profit does not depend on r. To explain, when the licensing firm sets its price, it takes into account the per unit licensing fee, r. $r(1-x)$ is the profit from selling its own technology to the rival firm. When the licensing firm increases its output level (x), the rival's supply $(1-x)$ decreases and then the profit from licensing $r(1-x)$ also decreases. The licensing fee can be interpreted as the opportunity cost to the licensing firm.[9]

[9] Note that the result that the quantities supplied by the firms do not depend on the level of the licensing fee r is different from that in Poddar and Sinha (2004). Using the Hotelling model with cost asymmetry, they

Since Firm 1's unit cost (the real cost plus the opportunity cost) is $c - d + r$ and Firm 2's unit cost is $c - d + r$, both firms face the same unit cost, which leads to equal market share. An increase in r increases per unit cost of each firm and raises equilibrium prices. This is why the profit of Firm 2 does not depend on r.

This property is similar to that in Sappington (2005). Using a simple duopoly model, he considers the problem of access charge and the entrant firm's make-or-buy decision. The vertically integrated incumbent firm charges the input price w to the entrant firm. He shows that the level of w does not affect the quantities supplied by the firms.[10]

Since $r \leq d$, Firm 1 naturally sets $r = d$. Since an increase in r does not reduce the profit of Firm 2 and increases that of Firm 1, Firm 1 chooses the maximal royalty fee and Firm 2 has no reason to reject it. Note that this result does not depend on the distribution of bargaining power between Firms 1 and 2.

4 Result: Endogenous locations

We now model endogenous firm locations. Before we solve the problem, we show a closely related result provided by Ziss (1993). Using the Hotelling model with cost asymmetry, he shows that pure strategy equilibria do not exist if and only if $d \geq (6 - 3\sqrt{3})t$ (i.e., the cost difference between the firms d is large enough). We now show that when the efficient firm is able to license its efficient technology to the inefficient rival, in all cases, maximum differentiation appears in equilibrium. Obviously, the location equilibrium is quite different from that in Ziss (1993).

consider the optimal licensing strategy of both an outsider as well as an insider patentee. Although Poddar and Sinha (2004) present several interesting insights about optimal licensing strategies, they do not consider the effect of the licensing fee on pricing strategy. Therefore, in their paper, the quantity supplied by the licensed firm is $(3 - r)/6$, where r is a royalty rate (Poddar and Sinha 2004, p. 215). In our paper, we assume the patentee is an insider (Firm 1 holds the patent). If we consider an outsider patentee, we can show that the patentee's profit is d, which is exactly the same as the additional profit of insider patentee. This implies that neutrality (both insider and outsider patentees have the same incentive for R&D) holds in our spatial model.

[10] Note that the main proposition of Sappington (2005) is not the property already mentioned, but the following one: regardless of the established price (w) of the upstream input, the entrant prefers to make (*resp.* buy) the upstream input itself (*resp.* from the incumbent) when it (*resp.* the incumbent) is the least-cost supplier of the input.

To derive the location equilibrium explicitly, we assume that the transport costs of consumers are quadratic in distance. The assumption is similar to that in Ziss (1993). That is:

$$u_x = \begin{cases} -t(x_1 - x)^2 - p_1 & \text{if bought from Firm 1,} \\ -t(1 - x_2 - x)^2 - p_2 & \text{if bought from Firm 2.} \end{cases} \tag{5}$$

For a consumer living at:

$$x = \frac{1 - x_1 - x_2}{2} + \frac{p_2 - p_1}{2t(1 - x_1 - x_2)}, \tag{6}$$

total utility is the same no matter which of the two firms is chosen. Given the values of x_1, x_2, r, the profits of the firms are:

$$\begin{aligned} \pi_1 &= (p_1 - (c - d))x + r(1 - x) \\ &= (p_1 - (c - d) - r)\left(\frac{1 - x_1 - x_2}{2} + \frac{p_2 - p_1}{2t(1 - x_1 - x_2)}\right) + r, \qquad (7) \\ \pi_2 &= (p_2 - (c - d) - r)(1 - x) \\ &= (p_2 - (c - d) - r)\left(\frac{1 + x_1 + x_2}{2} + \frac{p_1 - p_2}{2t(1 - x_1 - x_2)}\right). \qquad (8) \end{aligned}$$

The first-order conditions lead to:

$$\begin{aligned} p_1 &= (c + r - d) + \frac{t(1 - x_1 - x_2)(3 + x_1 - x_2)}{3} \\ p_2 &= (c + r - d) + \frac{t(1 - x_1 - x_2)(3 - x_1 + x_2)}{3}. \end{aligned}$$

Substituting the prices into the profit functions in (7) and (8), we have:

$$\begin{aligned} \pi_1 &= \frac{t(1 - x_1 - x_2)(3 + x_1 - x_2)^2}{18} + r, \\ \pi_2 &= \frac{t(1 - x_1 - x_2)(3 - x_1 + x_2)^2}{18}. \end{aligned}$$

For the same reason discussed in the previous section, Firm 1 sets the level of r at $r = d$ in the second stage. Differentiating π_i with respect to x_i, we have:

$$\begin{aligned} \frac{\partial \pi_1}{\partial x_1} &= -\frac{t(1 + 3x_1 + x_2)(3 + x_1 - x_2)}{18} < 0, \\ \frac{\partial \pi_2}{\partial x_2} &= -\frac{t(1 + x_1 + 3x_2)(3 - x_1 + x_2)}{18} < 0. \end{aligned}$$

The optimal locations of the firms are $x_1 = x_2 = 0$. That is, the maximum differentiation appears in equilibrium. The profits of the firms are:

$$\pi_1 = \frac{t}{2} + d, \quad \pi_2 = \frac{t}{2}.$$

When we take into account licensing activity, the well-known maximum differentiation result appears even though the cost differential between the firms is significant. The result is quite different from that in Ziss (1993).

To discuss the difference between this model and that in Ziss (1993), we consider the mixed strategy equilibria in Ziss (1993). For the nonexistence problem, we can show that two firms independently choose two edges of the linear city with equal probability if $d \geq (6-3\sqrt{3})t$ (this is a sufficient condition). Under the mixed strategy equilibrium, the following four location patterns appear as the firms' location choices: (1) Firm 1 locates at 0 and Firm 2 locates at 1 (with probability 1/4); (2) Firm 1 locates at 1 and Firm 2 locates at 0 (with probability 1/4); (3) Firm 1 locates at 0 and Firm 2 locates at 0 (with probability 1/4); and, (4) Firm 1 locates at 1 and Firm 2 locates at 1 (with probability 1/4).

Under the former two patterns (the maximum differentiation, with probability 1/2), the profits of the firms are as follows:

$$\pi_1 = \begin{cases} \dfrac{(3t+d)^2}{18t} & d < 3t, \\ d - t & d \geq 3t, \end{cases} \qquad \pi_2 = \begin{cases} \dfrac{(3t-d)^2}{18t} & d < 3t, \\ 0 & d \geq 3t. \end{cases} \tag{9}$$

Under the latter two patterns (the minimum differentiation at the edge, with probability 1/2), the profits of the firms are as follows:

$$\pi_1 = d, \quad \pi_2 = 0. \tag{10}$$

When $d \geq (6 - 3\sqrt{3})t$, given that the firms employ the above-mentioned mixed strategies, the expected profits of the firms are:

$$\pi_1 = \frac{(3t+d)^2}{36t} + \frac{d}{2}, \quad \pi_2 = \frac{(3t-d)^2}{36t}. \tag{11}$$

For any $d < 3t$, profits are smaller than those in which the efficient firm licenses its technology to the inefficient rival.

5 Main result: R&D investments and location choice

To show the main result of our paper, we now consider the following four-stage game: first, each firm chooses its location in a linear city; second, each firm chooses whether or not it engages in R&D investment that probabilistically affects its production cost; third, one of the firms that has a cost advantage over its rival chooses whether or not it licenses its technology to the rival, and chooses the royalty fee when it licenses; and fourth, each firm sets its price.[11]

We now assume that Firm 1 has a cost advantage after its investment in R&D and that the difference between the two firms' marginal costs is d. We have already shown that, given locations and marginal costs, if the advantaged firm licenses its technology, the profits of the firms are:

$$\begin{aligned}\pi_1^L &\equiv \frac{t(1-x_1-x_2)(3+x_1-x_2)^2}{18}+d,\\ \pi_2^L &\equiv \frac{t(1-x_1-x_2)(3-x_1+x_2)^2}{18}.\end{aligned}$$

If the advantaged firm does not license its technology, the profits of the firms are:

$$\begin{aligned}\pi_1^N &\equiv \begin{cases} d-t(1-x_2-x_1)(1-x_1+x_2), & \text{if } d \geq t(1-x_2-x_1)(3-x_1+x_2),\\ \dfrac{(d+t(1-x_1-x_2)(3+x_1-x_2))^2}{18t(1-x_1-x_2)}, & \text{if } d < t(1-x_1-x_2)(3-x_1+x_2),\end{cases}\\ \pi_2^N &\equiv \begin{cases} 0, & \text{if } d \geq t(1-x_1-x_2)(3-x_1+x_2),\\ \dfrac{(t(1-x_1-x_2)(3-x_1+x_2)-d)^2}{18t(1-x_1-x_2)}, & \text{if } d < t(1-x_1-x_2)(3-x_1+x_2).\end{cases}\end{aligned}$$

In any case, the advantaged firm has the incentive to license its technology for its rival in the third stage. In the first and the second stages, each firm anticipates the profit functions π_1^L and π_2^L.

The results in the second stage only affect on the value of d in π_1^L. The locations are independent from the results in the second stage that are related to the value of d. The equilibrium locations are similar to those in Section 4. Therefore, maximum differentiation

[11] Again, the order of the license fee decision and location choice stages is interchangeable. Our result holds if the location choices are made after the determination of r.

appears in equilibrium even though each firm undertakes R&D. The result is quite different from those of Gerlach *et al.* (2005) and Christou and Vettas (2005).

6 Further discussion on R&D investments

We now explicitly introduce R&D decisions into the basic model. The unit cost of the product for each firm is c_i ($i = 1, 2$), which is determined by its investment. Each firm chooses whether to invest to reduce its own production costs. By its investment, firm i has to incur the fixed cost of the investment F, and it reduces its marginal production cost at $c_i = c - h$ ($h \in [0, \bar{c}]$). h depends on a density function $f(h)(\geq 0)$ that has the following property: $F(h) \equiv \int_0^h f(t)dt$ and $F(\bar{c}) = 1$. We assume that $f(h)$ and $F(h)$ are continuous and differentiable functions. After the firms invest, Nature determines c_1 and c_2 independently and simultaneously

We can summarize the situation with the following payoff matrix.

		Firm 2	
		I	N
Firm 1	I	$E[\pi_1(I,I)]$, $E[\pi_2(I,I)]$	$E[\pi_1(I,N)]$, $E[\pi_2(I,N)]$
	N	$E[\pi_1(N,I)]$, $E[\pi_2(N,I)]$	$E[\pi_1(N,N)]$, $E[\pi_2(N,N)]$

To check the incentive to invest, we have to evaluate:

$$(1)\ \Delta_1 \equiv E[\pi_1(I,N)] - E[\pi_1(N,N)], \quad (2)\ \Delta_2 \equiv E[\pi_1(I,I)] - E[\pi_1(N,I)].$$

Δ_1 is the expected additional profit from the investment given that the rival firm does not invest. Δ_2 is the expected additional profit given that the rival firm does invest. If $\Delta_i \geq F$, the firm has the incentive to invest given that the $i-1$ firm invests.

Case (1): Profit As discussed earlier, if a firm has a cost advantage, it has the incentive to license its own advanced technology for its rival. The firm earns additional profit d from the license, where d is the difference between the marginal costs of the firms. On the other hand, the profit of a firm using the licensed technology is constant, $t/2$. Under Case 1, the expected additional profit of the investing firm (Δ_1^a) is:

$$\Delta_1^a = \int_0^{\bar{c}} \left(v + \frac{t}{2}\right) f(v)dv - \frac{t}{2} = \int_0^{\bar{c}} vf(v)dv.$$

Case (1): Welfare We now suppose that the social planner anticipates the licensing regime. Regardless of the results of R&D activities, the quantity supplied by each firm is 1/2 and the prices set by the firms are the same. Therefore, R&D only affects the production costs of the firms. Under the licensing regime, the most efficient firm's technology is available for all firms. For instance, when an innovating firm's marginal cost becomes $c - h$, then the other firm's marginal cost also becomes $c - h$ and the social benefit of the R&D is $h \times 1 = h$ (1 is the total quantity supplied by the firms). If one firm invests, the gross benefit of the R&D investment is:

$$\Delta W_1^a \equiv \int_0^{\bar{c}} v f(v) dv.$$

Comparison (1) The difference between Δ_1^a and ΔW_1^a is 0. The incentive for R&D investments by the firms is consistent with the additional social benefit of that R&D. Socially efficient R&D investment is achieved by the firms without any governmental intervention.

As mentioned earlier, when an innovating firm's marginal cost becomes $c - h$, then the other firm's marginal cost also becomes $c - h$ and the social benefit of the R&D is $h \times 1 = h$. By licensing, the innovating firm can take the whole social benefit of the R&D, h. Therefore, the incentive for R&D investment by the firm is consistent with the additional social benefit of that investment. This logic can be also applied to the following case.

Case (2): Profit The more efficient firm earns additional profit d from the license, where d is the difference between the marginal costs of the firms. Under Case 2, the expected additional profit of the second investing firm (Δ_2^a) is:

$$\begin{aligned} \Delta_2^a &= \int_0^{\bar{c}} \int_0^{\bar{c}} (v - \min\{v, t\}) f(t) dt f(v) dv \\ &= \int_0^{\bar{c}} \int_0^{v} (v - t) f(t) dt f(v) dv. \end{aligned}$$

$v - \min\{v, t\}$ is the level of the *ex post* cost advantage by the investment, where t is the level of the rival firm's cost reduction and v is the level of its own cost reduction. If $v > t$, the level is positive and the firm gains additional profit $v - t$, otherwise it is zero.

Case (2): Welfare If the most efficient innovating firm's marginal cost becomes $c-h$, then the other firm's marginal cost also becomes $c-h$ and the social benefit of the R&D is $h \times 1 = h$ (1 is the total quantities supplied by the firms). If both firms invest, the *gross* benefit of the R&D investment is:

$$\begin{aligned}\Delta W_2^a &= \int_0^{\bar{c}}\int_0^{\bar{c}} \max\{v,t\} f(t)dtf(v)dv \\ &= \int_0^{\bar{c}}\int_0^{v} vf(t)dtf(v)dv + \int_0^{\bar{c}}\int_v^{\bar{c}} tf(t)dtf(v)dv.\end{aligned}$$

$\max\{v,t\}$ is the level of the efficient firm's cost reduction. If $v > t$, the decrease in the social marginal cost is v, otherwise, it is t. To derive the *additional* benefit of the second R&D investment, we change the expression ΔW_1^a as follows:

$$\Delta W_1^a = \int_0^{\bar{c}}\left(\int_0^{v} vf(t)dt + \int_v^{\bar{c}} vf(t)dt\right) f(v)dv.$$

The additional social benefit of the second R&D investment is:

$$\Delta W_2^a - \Delta W_1^a \;=\; \int_0^{\bar{c}}\int_v^{\bar{c}} (t-v)f(t)dtf(v)dv.$$

Comparison (2) The difference between Δ_2^a and $\Delta W_2^a - \Delta W_1^a$ is:

$$\begin{aligned}\Delta_2^a - (\Delta W_2^a - \Delta W_1^a) &= \int_0^{\bar{c}}\int_0^{v} vf(t)dtf(v)dv + \int_0^{\bar{c}}\int_v^{\bar{c}} vf(t)dtf(v)dv \\ &\quad - \left(\int_0^{\bar{c}}\int_0^{v} tf(t)dtf(v)dv + \int_0^{\bar{c}}\int_v^{\bar{c}} tf(t)dtf(v)dv\right) \\ &= \int_0^{\bar{c}}\int_0^{\bar{c}} vf(t)dtf(v)dv - \int_0^{\bar{c}}\int_0^{\bar{c}} tf(t)dtf(v)dv \\ &= \int_0^{\bar{c}} vf(v)dv - \int_0^{\bar{c}} tf(t)dt \times \int_0^{\bar{c}} f(v)dv \\ &= 0.\end{aligned}$$

The incentive for R&D investment by the firms is consistent with the additional social benefit of the R&D investment. Efficient R&D investment is achieved by the firms without any governmental intervention.

7 R&D investments with and without licensing

We now compare R&D incentives in the following two cases: (a) each firm licenses its technology whenever it has a cost advantage; (b) neither firm licenses its advanced technology.

The basic setting is similar to that in the former section, except for the following simplification. By its investment, firm i reduces its marginal production cost at $c_i = c - \bar{c}$ with probability p; $c_i = c - \bar{c}/2$ with probability q; $c_i = c$ with $1 - p - q$. After the firms invest, c_1 and c_2 independently and simultaneously. As mentioned in the former section, to check the incentive to invest, we now evaluate:

$$(1)\ \Delta_1 \equiv E[\pi_1(I,N)] - E[\pi_1(N,N)], \quad (2)\ \Delta_2 \equiv E[\pi_1(I,I)] - E[\pi_1(N,I)].$$

Δ_1 is the expected additional profit from the investment given that the rival firm does not invest. Δ_2 is the expected additional profit given that the rival firm invests. If $\Delta_i \geq F$, the firm has the incentive to invest, given that $i - 1$ firm invests.

Case (1) Given that no firm invests in R&D activity, the difference between the (gross) marginal gains in the cases (a) and (b) is (note that $c < 3t$):

$$\frac{\bar{c}(4(12t - \bar{c})p + (24t - \bar{c})q)}{72t} > 0. \tag{12}$$

In any case, the R&D incentive under the licensing case is stronger than that under the non-cooperative case.[12] The reason is simple. When the innovating firm licenses its technology, it can earn the whole additional social gain. On the other hand, when the innovating firm does not license its technology, it can earn at most the product of its quantity supplied and the level of the reduction in its marginal cost.

Case (2) Given that one of the firms invests in R&D activity, the difference between the (gross) marginal gains in cases (a) and (b) is:

$$\frac{\bar{c}(12[2(2p+q) - 3(2p^2 + 2pq + q^2)]t - (4p + q - 2(2p+q)^2)\bar{c})}{72t}. \tag{13}$$

The sign of the fraction depends on the values of p, q, t, and h. Figure 1 depicts the regions in which the sign is negative and positive when $\bar{c} = t/10$.[13] When $p + q$ is not large enough,

[12] This result does not depend on the distribution function of the success probability of R&D investments $f(h)$.

[13] The shape does not depend very much on the value of $\bar{c}/t$.

the R&D incentive under the licensing case is stronger than that under the noncooperative case.

We consider a case in which $p + q$ is large enough, that is, the probability of success in R&D is large enough. In the licensing case, given the rival firm invests, the probability that the other firm has a cost advantage over its rival is small, because the probabilities of success are high enough to allow both firms to succeed in their R&D investments. For instance, if the marginal cost of an investing firm is $c - d$ with probability 1, due to the investment, the additional benefit of the second R&D investment is *zero*, that is, the additional R&D investment is meaningless. In the nonlicensing case, given the rival firm invests, if the other firm does not invest, it has a cost disadvantage with higher probability. Contrary to the licensing case, the cost disadvantage diminishes the profit of the inefficient firm. To avoid being the inefficient firm, each firm has a strong incentive to invest. For instance, if the marginal cost of an investing firm is $c - d$ with probability 1, due to the investment, the additional benefit of the second R&D investment is $t/2 - (3t - d)^2/18 = d(6t - d)/18$ (see Section 4).

8 Concluding remarks

We have investigated the relationship between licensing activities and equilibrium locations in a product differentiation model. We also discuss the welfare implication of R&D investment. We show that the level of royalty fee does not affect equilibrium locations nor equilibrium profit of licenser and that firms choose the maximal license fee. We show that licensing activities after R&D always lead to maximum differentiation between firms and mitigate price competition. Furthermore, we show that those licensing activities induce socially optimal R&D.

Our results have a testable implication for the literature of licensing. We have shown that an efficient firm has an incentive to transfer its advanced technology to its rival and that the transfer mitigates competition between these firms. In the setting, we have implicitly assumed that the technology is transferable. On the other hand, we think that the results derived by Gerlach *et al.* (2005) and Christou and Vettas (2005) are suitable for industries in which technological transfers are difficult or not available. That is, spatial agglomeration that leads to tough competition appears if technological transfers are difficult.

Given our results, we think that the following is an important research question: are the price–cost margins of firms positively related to technological transferability, which, in turn, is positively related to propensities to license? If technologies are transferable, price competition is moderate and firms are able to earn greater than normal profits. On the other hand, if technologies are not transferable, tough competition appears and firms earn normal or subnormal profits. Future research is required to test this problem.

In this paper, we assume that the licenser can transfer its advanced technology without any transaction costs. However, some technologies are difficult to transfer, and firms can choose whether they invest in nontransferable or transferable technology. We think that competitive structure affects this choice and this is an important problem in the context of R&D investment. This remains a topic for future research.

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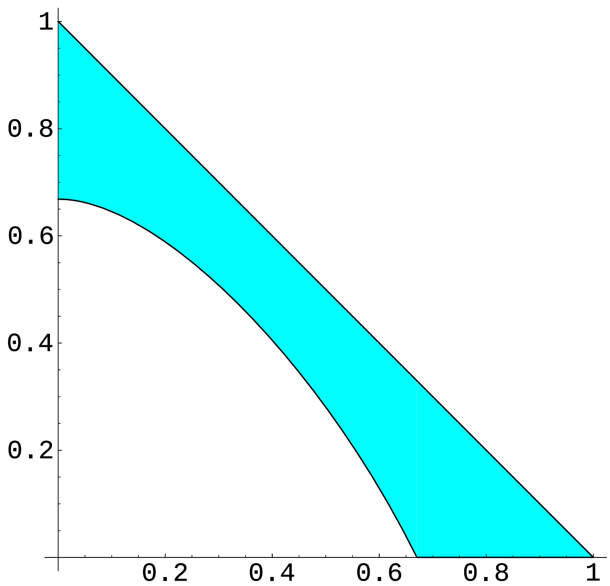

Figure 1: R&D incentives

Horizontal: p, Vertical: q, The shaded area: $\Delta_1^a - \Delta_1^b < 0$, The white area: $\Delta_1^a - \Delta_1^b > 0$

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