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Estimation of Optimal Metropolitan Size in Japan with Consideration of Social Costs

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by

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[Abstract]: The main purpose of this study is to estimate the optimal city size which would attain maximum total surplus and sustainability, or a city size in which total benefits would equal total costs. We apply regressions to the total benefit function and the total cost function for 269 employment metropolitan areas for the year 2000 in Japan. Our study can be distinguished from others in that we include in total costs such social costs as environmental pollution. Our findings are that the optimal city size is 393,151 persons. The sustainable limit for city size is 1,057,412.

[JEL Classification]: R10, R12, Q50
[Key Words]: Optimal City Size, Total Surplus, Environmental Costs, Social Costs

1. Introduction

Urban economists have long been interested in determining the optimal size of a city. Theoretical studies on optimal city size include those by Alonso (1971), Henderson (1974a), Arnott (1979), and Kanemoto (1980). Optimal city size is defined as that which maximizes the potential welfare of participants in the economy. Henderson (1974a) determines equilibrium and optimal city size by presenting a general equilibrium model of a city where production and consumption occur. Henderson (1974b) investigates whether market-achieved city size is greater or less than optimal city size when externalities such as air pollution are considered. Arnott (1979) contributes theoretical work that includes spatial considerations and a utility-maximizing framework. Fujita (1989) offers sound theoretical foundations for optimal city size. Although definitions vary in their details (e.g. see Nakamura and Kaneuchi (2001)), in essence optimal city size means that attaining the maximum difference between the aggregate total benefits and total costs of a city.

Compared with the theoretical work that has been done on optimal city size, empirical studies have been less developed and seem to provide inconsistent results. These empirical studies can be divided into three groups.[1] The first group seeks optimal city size from several perspectives, to determine the relationship between city size and urban agglomeration economies. For example, Kelly (1977) estimates the relationship between wage, employment and urban dimensions such as city population, urbanized population, and population density. Although not an empirical study, Moomaw's (1981) work reevaluates previous studies concerning the relationship between productivity and city size. And Henderson (1986) investigates the relationship between efficient resource usage and city size by using data from SMSA (Standard Metropolitan Statistical Area) in the U.S. Yezer and Goldfarb (1978) develop a model demonstrating the conditions necessary for efficient distribution of labor across areas under the existence of agglomeration economies and congestion. They also apply an indirect empirical test for efficient allocation. After estimating the relationship between city size and wage in 90 U.S. cities, they conclude that there are specific city size ranges where necessary conditions for efficient allocation of resources are not met, particularly for cities in the 1.5 to 2.5 million population range.

The second group of empirical studies evaluates optimal city size by using the Henry George Theorem. Kanemoto et al. (1996) summarize the essence of the theorem as follows: If the only agglomeration forces are the commuting costs of workers who work at the center of the city, then the optimal city size is achieved when the Pigouvian subsidy for the agglomeration externalities

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[1] Empirical studies related to city size include the following: an evaluation of Chinese city size by Au and Henderson (2006); an exploration of the relationship between labor productivity and such structural features of a city as its size, sprawl and traffic speed by Prud'homme and Lee (1999); an investigation into the effect of a compact city by Burton (2000); a study of factors associated with worker productivity by Cervero (2001); and a study of the compact city as a sustainable urban form by Holden and Norland (2005).

equals the total differential urban rent. There are two studies in this group: Kanemoto et al. (1996) and Kanemoto and Saito (1998). Kanemoto et al. (1996), basing their evaluation on the Henry George Theorem, empirically investigate whether or not Japanese cities, especially Tokyo, exceed optimal city size. They estimate aggregate production functions for metropolitan areas in Japan in order to derive the magnitudes of agglomeration economies. Because of the difficulty of converting land prices into land rents, Kanemoto et al. do not test the Henry George Theorem directly. Instead, they compare the ratio of the total land values to the total Pigouvian subsidies for each metropolitan area, and they conclude that there is no evidence supporting the hypothesis that Tokyo is too large. Kanemoto and Saito (1998)'s study also evaluates whether or not Tokyo is too large by using the same methodology as Kanemoto et al. (1996). However, this study differs from the previous one in the definition of metropolitan areas and in the estimation method of total land values. Kanemoto and Saito (1998) conclude from their results that Tokyo is too large, an outcome opposite to Kanemoto et al. (1996).

The third group of empirical studies employs an approach that estimates benefit and cost functions. Studies in this group include Capello and Camagni (2000), Nakamura and Kanauchi (2001), and Zheng (2007). Capello and Camagni (2000)'s real purpose is to take a critical view of theoretical works on city size. In their study, they use 58 Italian cities, estimating the average location benefit function and the average location cost functions, which consist of variables such as the size of city (population), the type of urban functions developed, and the network integration level. They show that the city size attaining the highest average location benefit and the lowest average location cost is 361,000 and 55,500 population, respectively. Nakamura and Kanauchi (2001) investigate optimal city size by using Japanese city data for 1975, 1980, 1985, 1990, 1995 and 1997. They select observations of between 666 and 693 cities for each year and estimate both the average benefit function and the average cost function. In their study, they define several kinds of city size, including the social optimal city size, the market equilibrium city size, the minimum cost city size, and so on. Among the authors' several findings are that the social optimal city size in Japan is between 3.32 million to 5.21 million population, and that the magnitude of the social optimal city size declined between 1975 and 1997. Finally, Zheng (2007) estimates optimal city size by using 43 Japanese metropolitan areas in the year 2000. He constructs urban theoretical models and estimates both the total benefit and total cost functions based on the models. Total benefit is defined as total disposal income, and total cost is defined as total expenditures of households. The optimal city is defined as the maximum total surplus, which is the difference between total benefits and total costs. Zhang concludes that a city of optimal size has a population of about 18 million.

While previous empirical studies have made important contributions, they have been insufficient not only in their number but also in their failure to consider important factors. First, many empirical studies do not have theoretical foundations. There are no explanations as to why certain variables are selected.

Second, empirical results are inconsistent, with optimal size defined by the same measures varying from study to study. Nakamura and Kaneuchi (2001)'s results show that the optimal city has a population of between 3.32 million and 5.21 million, but Zheng (2007)'s optimal city has 18 million people.

Last, and most importantly, there are no studies in which social costs such as environmental pollution have been skillfully taken into account. More attention should be paid to social costs. It is the purpose of this study to consider such costs as we estimate optimal city size.

As for our empirical approach, we follow Nakamura and Kaneuchi (2001) and Zheng (2007). As Arnott (2004) implies, Kanemoto et al. (1996)'s indirect approach of using the Henry George Theorem still has weaknesses and is difficult to execute in empirical investigation.

The basic framework of our study follows the pioneering work of Zheng (2007), but our model differs from Zheng's in several ways. First, we include the social cost of environmental pollution in the model. Second, we consider the public sector (local government), which provides public goods to reduce the city's environmental pollution. Third, we set up a model with more general function forms, which are different from those of Zheng (2007). Fourth, we do not include real estate providers. Last, we use a greater sample size, one that includes smaller metropolitan areas. Despite these differences, our study has similarities to Zheng's. First, the empirical models are estimated by the function of population, with other parameters such as price of output, housing price and utility level being fixed over metropolitan areas. Second, using Japanese city data, we estimate both the benefit and cost functions of the city. In Japan, as there is no definition of *metropolitan area*, we opt to use the metropolitan employment areas defined by Kanemoto and Tokuoka (2002). We use data from the year 2000 in estimating optimal city size.

2. Model of Optimal City

2.1 Basic Structure

Before we explain the structure of the city model, we will summarize important characteristics and basic assumptions. First, in this model, we consider three important actors: the firm, the household, and the local government. The firm produces the single exported good. In order to produce the good, the firm employs labor from the household, which receives wages from the firm and consumes the imported good, as well as housing. The household must remit tax to the local government. During the course of certain of these activities, both the firm and the household generate environmental pollution, the negative effects of which decrease the household's utility level. The local government's role is to provide for the public good, in this case by reducing environmental pollution, but it must do so under the budget constraints of tax revenues.

2.2 Individual Sector

(1) Firm

A firm is producing the exported good, Q_X, by using one input factor, population as labor N. The firm decides the amount of labor (population) required according to how it can maximize its profit. The wage rate at the maximum, w^*, can be obtained as a function as follows:

$$Q_X = Q_X(N) \tag{1}$$

$$\max_N \quad p \cdot Q_X(N) - w \cdot N \tag{2}$$

$$w^* = p \cdot Q'_X(N) \tag{3}$$

where $Q_X(.)$: production function,

Q_X: exported product,

N: population,

p: price of the exported product,

w: wage.

(2) Household

A household's utility is obtained by consuming the imported good, z, and a house, s, but its utility decreases when the environmental condition, e, worsens. The household decides the amount of imported good and house according to how it can maximize its utility under budget constraints.

$$U = U(z, s) - e \tag{4}$$

$$p \cdot z + r \cdot s = w - t, \tag{5}$$

where $U(.)$: utility function,

z: imported goods,

s: housing,

e: environment,

p: price of imported goods

r: price of housing,

t: tax.

Given the utility level, the optimization problem is to determine how the household can minimize its expenditure when deciding how much imported good and housing it can consume.

$$\min_{z,s} p \cdot z + r \cdot s + t \tag{6}$$

$$\text{s.t} \quad U = U(z, s) - e \tag{7}$$

By minimizing the household's expenditure function, we can obtain each good's compensated demand function as follows:

$$z^* = z(p, r, U, e) \tag{8}$$

$$s^* = s(p, r, U, e). \tag{9}$$

By substituting z* and s* into the household's expenditure, $p \cdot z + r \cdot s$, we can obtain the following expenditure function:

$$HC^* = p \cdot z\,(p, r, U, e) + r \cdot s\,(p, r, U, e) = HC\,(p, r, U, e). \tag{10}$$

(3) Externalities: Environmental Pollution

Environmental pollution as negative externality is considered. Environmental conditions deteriorate as both the firm's production and the population increase. The function of environmental pollution in a city is expressed as follows:

$$C_E = C_E\,(Q_X, N), \tag{11}$$

where C_E: environment pollution.

This equation can be rewritten from equation (1):

$$C_E = C_E\,(Q_X\,(N), N) = C_E\,(N)\,. \tag{12}$$

(4) Local Government

The role of local government is to provide for the public good by reducing the environmental pollution caused by the activities of both firms and households. The local government levies lump-sum tax on households, and total tax revenues are $t \cdot N$. The local government spends all tax revenues on public goods to reduce environmental pollution. Therefore, tax revenues equals production of public goods.

$$t \cdot N = q \cdot g \tag{13}$$

where q: price of public goods,

g : amount of public goods.

Environmental pollution after the production of public goods is expressed as follows:

$$e = C_E\,(N) - q \cdot g = C_E\,(N) - t \cdot N = e\,(N, t). \tag{14}$$

By substituting equation (14) into equation (10), the following result is obtained:

$$HC^* = HC\,(p, r, U, e(N, t))$$
$$= HC\,(p, r, U, N, t). \tag{15}$$

2.3 Total surplus, Total Benefits and Total Costs of a City

Total benefits, TB, of a city are ultimately distributed to the total of households. The figure for household's total revenues is obtained as follows:

$$TB = w^* \cdot N$$
$$= p \cdot Q'_X\,(N) \cdot N. \tag{16}$$

On the other hand, the total cost, TC, of a city is the final expenditure of households and the environmental pollution level. It is obtained as follows:

$$TC = HC^* \cdot N + e$$

$$= HC\,(p, r, U, N, t) \cdot N + e\,(N, t). \quad (17)$$

Finally, total surplus, *TW*, is the difference between total benefits and total costs, and is expressed as follows:

$$TW = TB - TC$$
$$= p \cdot Q'_X\,(N) \cdot N - HC\,(p, r, U, N, t) \cdot N - e\,(N, t). \quad (18)$$

2.4 Empirical Model

In the previous section, we saw that by estimating equation-(16), (17) and (18), we can obtain total benefits, costs, and surplus. In this study, we estimate total benefits (*TB*) and total costs (*TC*) separately. First, total benefit is a function of the price of exported output (*p*), labor productivity (Q'_X (*N*)), and population (*N*). Although labor productivity itself is a function of population, *N*, productivity level might be affected by the infrastructure of a city, such as in relation to transport conditions. In fact, an empirical study by Cervero (2001) shows a positive relation. Therefore, we include transport conditions as control variables. From the sample distribution, in this study, this function is specified as the log-linear function:

$$ln\,(TB/p) = \alpha + \beta\, ln\, N + \mu_1 lnTR_1 + \mu_2 lnTR_2 \quad (19)$$

where *TB*: total benefit

p: price of exported output

N: population of a city

TR_1: road density

TR_2: ratio of public transportation.

Second, total cost is a function of household expenditure (*HC*), population (*N*), and environmental pollution (*e*). Household expenditure is a function of the price of exported output (*p*), housing price *(r)*, utility level (*U*), population (*N*), and tax rate *(t)*. Furthermore, environmental pollution is a function of population (*N*) and tax rate *(t)*. In this study, we assume that only utility level is constant among different metropolitan areas. Therefore, total cost function is specified as the log-linear function as follows:

$$ln\,(TC) = \delta + \gamma\, ln\, N + \pi\, ln\, p + \rho\, ln\, r + \tau\, ln\, t \quad (20)$$

where *TC*: total costs

p: price of exported output

N: population of a city

r: housing price

t: tax rate.

As a result, we estimate the functions shown in equation (19) and (20) separately.

3 Data

3.1 Sample Selection

As for the analysis, data is collected based on metropolitan areas. However, as there is no official definition for metropolitan area in Japan, this study uses the definition of Kanemoto and Tokuoka (2002), as do other studies such as Mizuno et al. (2006), and Zheng (2007). We use 269 employment metropolitan areas, consisting of 113 large employment metropolitan areas and 156 small metropolitan areas, as defined by Kanemoto and Tokuoka (2002). Data on these areas was collected in 2000.

3.2 Definition of Variables

(1) Total Benefits

First, total benefits (*TB*) are defined as the total incomes of a city. One benefit is one individual income because final benefits are received by each individual. The figure for the city's total incomes is obtained by multiplying the average individual income of a city by its population. It is worth noting that commuting subsidies from companies are excluded from average individual income, as in Zheng (2007). Details of variables are explained as follows.

Total benefits are defined as total annual incomes after subtracting commuting subsidies:

$$TB = AR - COM \qquad (21)$$

where *TB* : total benefits

AR : total annual incomes

COM : commuting subsidies.

Total annual incomes are obtained as follows:

$$AR = (AHR / HP) \cdot N \qquad (22)$$

where *AHR* : annual incomes per household

HP : persons per household

N : city population.

For some cities, data are not available for total annual incomes per household. Therefore, we estimate total annual incomes by using taxable incomes. The estimated results are as follows:

$$AR/N = 1217.31 + 0.747\ (YT/N) \qquad (23)$$

$$(14.532)\ (11.010) \qquad R^2 = 0.205$$

where *YT*: taxable incomes

( ): t-statistics

R^2: coefficient of determination

sample size: 471.

Because the portion of total annual incomes that must be estimated accounts for less than 10% in a metropolitan area, we consider it acceptable to supply missing data by using this method.

We estimate commuting subsidies, following Zheng (2007):

$$CMS = CMSA \cdot [(ARU)^{1/2} / (ARAU)^{1/2}] \cdot L, \quad (24)$$

where *CMS*: commuting subsidies,
CMSA: average commuting subsidies per person,
ARU: habitable areas in metropolitan area,
ARAU: average habitable areas in metropolitan area,
L: number of employees.

Population figures are obtained from *National Census*, annual incomes per household and person per household are obtained from *National Survey of Family Income and Expenditure* (*Zenkoku Shohi Jittai Chosa*), and taxable income, habitable areas and number of employees are obtained from *the Municipality Basic Data (Shikuchouson Kiso Deta)*, all issued by the Ministry of Internal Affairs and Communications. Information on average commuting subsidies per person is obtained from *the General Survey on Working Conditions (Chingin Jijoutou Sogo Chosa)*, issued by the Ministry of Health, Labor and Welfare.

(2) Total Costs

Total costs are defined as the sum of household expenditures and environmental pollution costs as social costs:

$$TC = HC + SC \quad (25)$$

where *TC* : total benefits
HC : household expenditure
SC : environmental pollution costs.

Household expenditures consist of general consumption expenditures and housing expenditures.

$$HC = CNS + HEX \quad (26)$$

where *CNS* : general consumption expenditure
HEX : housing expenditures.

General consumption expenditures and housing expenditures are obtained as follows:

$$CNS = 12\,(MCNS / HP) \cdot N \quad (27)$$

where *MCNS* : monthly general consumption expenditure per household

HP : persons per household

N : city population

$$HEX = PLH + PRP \tag{28}$$

where *PLH* : repayment for land and housing

PRP : purchasing payment for properties.

Again, for some cities, data are not available on monthly general consumption expenditure per household. Therefore, we estimate general consumption expenditure by using annual commercial sales. The estimated results are as follows:

$$\ln (CNS/N) = 6.729 + 0.034 \ln (COM/N) \tag{29}$$

(57.219) (2.031) $R^2 = 0.009$

where *COM*: annual commercial sales

( ): t-statistics

R^2: coefficient of determination

sample size: 471.

Because the portion of general consumption expenditures that must be estimated accounts for less than 10% in a metropolitan area, we consider it acceptable to supply missing data by using this method.

Again, information on population is obtained from *National Census*. Monthly general consumption expenditures per household and person per household are obtained from *National Survey of Family Income and Expenditure* (*Zenkoku Shohi Jittai Chosa*), and annual commercial sales figures are obtained from *the Municipality Basic Data (Shikuchouson Kiso Deta),* all issued by the Ministry of Internal Affairs and Communications.

Environmental pollution costs consist of the social costs of industries and the social costs of transportation.

$$SC = INSC + TRSC \tag{30}$$

where *SC* : environmental pollution costs

INSC : social costs of industries

TRSC : social costs of transportation.

The social costs of industries result from air pollution. The social costs of CO_2, NO_x, SO_x and SPM emissions are obtained as follows:

$$INSC_{ij} = b_i \cdot \Sigma_j (\alpha_{ij} \cdot Y_j) \tag{31}$$

where $INSC_{ij}$: social costs of industry-*j* for emission type-*i*

b_i : unit cost of emission type-*i*

α_{ij} : coefficient of emission type-*i* in industry-*j*

Y_j : output of industrial products in industry-j

i : 4 kinds of emissions (CO_2, NO_x, SO_x and SPM)

j : 11 industries (agriculture and fishing, mining, construction, utilities, commerce and food, finance and insurance, real estate, transport and telecommunications, services, public service).

Information on the output of industrial products in industry-i is obtained from *Annual Report on Prefectural Accounts (Kenmin Keizai Keisan Nenpo)*, issued by the Cabinet Office. However, the statistics are based on prefectural units. Therefore, we allocate each prefecture's industrial product according to the proportion of the metropolitan areas' number of employees. The coefficient of emission type-i is obtained from *Embodied Energy and Emission Intensity Data for Japan Using Input*―*Output Tables (Sangyo Renkanhyo ni yoru Kankyo Fuka Gentani Deta Bukku*, issued by the National Institute for Environmental Studies.

The social costs of transportation consist of air pollution costs and congestion costs, obtained as follows:

$$TRSC_i = b_i \cdot \beta_i V + \gamma \cdot RL \qquad (32)$$

where $TRSC_{ij}$: social costs of transportation for emission type-*i*

b_i : unit cost of emission type-*i*

β_i: coefficient of emission type-*i* for travel length

V : total travel length by cars

γ : congestion costs per route-km

RL : route-km of roads

i : 4 kinds of emissions (CO_2, NO_x, SO_x and SPM).

Figures on total travel length by cars are obtained with the following equation:

$$V = 365\ TRP \cdot AREA \cdot ATRL \qquad (33)$$

where TRP: number of trip generations and attractions per area per day

$AREA$: total areas of a city

$ATRL$: average trip length per trip.

The coefficient of emission type-*i* for travel length is obtained from Namikawa et al. (2003). Figures for trip generation and attractions per area per day, and average trip length are obtained from *Road Transport Census (Doro Kotsu Sensasu)*, by the Ministry of Land, Infrastructure and Transport. To calculate these numbers, we assume that average traffic speed is 35 km/h. Finally, data for congestion costs per route-km are obtained from *Road Transport Census (Doro*

Kotsu Sensasu), by the Ministry of Land, Infrastructure and Transport. These values are adjusted according to the wage level of each region.

The unit cost of emission type-i is shown in Table 1.

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Table 1

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(3) Other Variables

First, population of a metropolitan area (*N*) is the sum of municipalities' populations, obtained from *National Census*.

Second, the price of exported goods (*p*) is the consumer price index for the general category, and housing price (*r*) is the consumer price index for the housing category. These are all obtained from *National Survey of Prices (Zenkoku Bukka Tokei Chosa)* issued by the Ministry of Internal Affairs and Communications.

Third, tax rate (*t*) is defined as general construction expenses per population. In this study, we assume that public expenditures are used for protection against environmental pollution. Therefore, general construction expenditures are considered to be encompassed in the concept of protection. Therefore, this variable is a kind of proxy variable. The figures for general construction expenditures are obtained from *Annual Statistics on Municipal Government Finance (Shi-cho-son betsu Kessan Jokyo Shirabe),* issued by the Ministry of Internal Affairs and Communications.

Road density (TR_1) is defined as total road length per habitable area. And ratio of public transportation (TR_2) is defined as the ratio of the number of employees commuting railway or bus to the total number of employees. These data are obtained from both *the Municipality Basic Data (Shikuchouson Kiso Deta)* and *National Census.*

(4) Statistics of Variables

Based on the definition of variables explained above, we construct data. All statistics used for the regression analysis are summarized in Table 2.

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Table 2

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4 Empirical Analysis of Optimal City Size

4.1 Regression Results

In this section, we apply regressions for the empirical models of total benefits and total costs, shown in equations (19) and (20). In addition to these models, we estimate the household expenditure function in order to compare the differences of results with the total cost function. The estimation method is the OLS. Regression results are summarized in Table 3, and the distribution of the actual total benefits and total costs are shown in Figure 1.

From these results, the goodness-of-fit in the regressions is acceptably high for these models because adjusted R^2 is very high: these are more than 0.98. Coefficients of these explanatory variables seem reasonable.

First, coefficients for population (N) show the positive sign. Therefore, both total benefits and total costs increase as population increases. However, the degree of the coefficient differs between the total benefit function, when it is less than one, and the total cost function, when it is more than one. These results indicate that at some degree of population, costs exceed benefits.

Second, as the transport conditions (i.e. road density (TR_1) and ratio of public transportation (TR_2)) improve, total benefits increase. However, road density is statistically significant but the ratio of public transportation is not. As small metropolitan areas are included in this study, most likely the ratio of public transportation, which has a small value in small metropolitan areas, becomes insignificant. Furthermore, the effect of road density to total benefits is not large.

Third, the price of exported output (p), housing price (r), and tax rate (t) all show the positive sign in the total cost function. Therefore, as these variables increase, total costs increase. Among these variables, the price of exported output and the housing price show larger effects on total costs. As these prices increase by 10%, total costs increase by about 4.7 to 5.3%.

Last, we compare the household expenditure function with the total cost function. The coefficient of population in the household expenditure function is one. This means that the household expenditure costs increase proportionally. Therefore, although at some point of population size household expenditures exceed total benefits, the point in this case would be larger than in the case of total costs.

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Table 3

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Figure 1

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4.2 Estimation of Optimal City Size and Sustainable Limit City Size

In this section, we estimate optimal city size and sustainable limit city size by using regression results. Optimal city population size is defined as that which attains the largest total surplus (*TW*). Total surplus (*TW*) of a city is obtained by subtracting a city's total costs (*TC*) from its total benefits (*TB*). That is, total surplus (*TW*) is obtained by *TW* = *TB* – *TC*, as shown in equation (17). In this study, we use models for total benefits *(TB*) and total costs (*TC)*, which are estimated separately. On the other hand, sustainable limit city population size is that in which a city's total benefits (*TB*) equal its total costs (*TC*). If a city exceeds the sustainable limit city size, it is deemed that the city's costs, including social costs, are larger than its total benefits. From a social point of view, it is not desirable for a city to exceed sustainable size.

Next, we would like to explain how to obtain the maximum city size and the sustainable limit city size empirically. As for the optimal city size, from both empirical models shown in equation (19) and (20), total surplus is obtained as follows:

$$TW = \mathrm{EXP}[\alpha + \beta \ln N + \mu_1 \ln TR_1 + \mu_2 \ln TR_2 + \ln p] - \mathrm{EXP}[\delta + \gamma \ln N + \pi \ln p + \rho \ln r + \tau \ln t]. \quad (34)$$

Strictly speaking, the total surplus of a city depends on these variables. However, our main focus is on the relationship between total surplus and city population size. When we consider total surplus a function of city population, the first order condition of maximization of total surplus attains the following condition:

$$\mathrm{EXP}[\alpha + \beta \ln N + \mu_1 \ln TR_1 + \mu_2 \ln TR_2 + \ln p]\,(\beta/\mathrm{N}) - \mathrm{EXP}[\delta + \gamma \ln N + \pi \ln p + \rho \ln r + \tau \ln t]\,(\gamma/\mathrm{N}) = 0. \quad (35)$$

From this equation, we can obtain the following equation on optimal city size (N^{op}):

$$N^{op} = \text{EXP}[(\alpha - \delta + \ln \beta - \ln \gamma + G_1 - G_2)/(\gamma - \beta)] \quad (36)$$

where $G_1 = \mu_1 lnTR_1 + \mu_2 lnTR_2 + \ln p$

$G_2 = \pi \ln p + \rho \ln r + \tau \ln t.$

As for the sustainable limit city size, as total benefits (TB) equal total costs (TC), from both empirical models shown in equation (19) and (20), the following condition holds:

$$\text{EXP}[\alpha + \beta \ln N + \mu_1 lnTR_1 + \mu_2 lnTR_2 + lnp] = \text{EXP}[\delta + \gamma \ln N + \pi \ln p + \rho \ln r + \tau \ln t]. \quad (37)$$

By rearranging this equation, we can obtain the following equation on sustainable limit city size (N^{sl}):

$$N^{sl} = \text{EXP}[(\alpha - \delta + G_1 - G_2)/(\gamma - \beta)] \quad (38)$$

where $G_1 = \mu_1 lnTR_1 + \mu_2 lnTR_2 + \ln p$

$G_2 = \pi \ln p + \rho \ln r + \tau \ln t.$

In the empirical results shown in Table 3, as γ is larger than β, the sustainable limit city size, N^{sl}, is larger than the optimal city size, N^{op}.

Next, we calculate both optimal city size and sustainable limit city size by using equations (36) and (38). These city sizes depend on the values of variables other than population, N. However, as we focus on city size, we fix values of these variables at the sample mean. Furthermore, the city size changes according to the magnitude of parameters, especially in the case of the coefficient for population. Therefore, we estimate the possible range of the optimal city size by changing the degree of coefficients for population. The estimation results of optimal city size and sustainable limit city size are shown in Table 4. Figure 2 shows the estimated total surplus.

First, the optimal city size, or that which attains the maximum surplus, is 393,151 persons. This population size characterizes the following metropolitan areas: Hakodate (367 thousand), Aomori (341 thousand), Takaoka (375 thousand) and Okazaki (379 thousand). As shown in Table 4, the size range is between 1,639,427 and 120,318 persons, much smaller than in previous studies, for example, a study by Zheng (2007), which shows that optimal city size is about 18 million. However, while Zheng’s study does not consider social costs such as environmental pollution, we take these costs into account, so that in our estimation, optimal city size becomes smaller. In fact, when we observe only social costs, we see that average social costs increase more sharply as population increases.

Second, the sustainable limit city population size, in which total benefits equal total costs, is 1,057,412. This kind of city size locates between 4,410,401 and 323,532. This kind of city size seems more close to the real metropolitan areas in Japan. According to our estimation, although the three largest metropolitan areas—Tokyo (31.8 million), Osaka (12.1 million), and Nagoya (5.4 million)—are too large, other metropolitan areas such as Kumamoto (1,021 thousand), Shizuoka (999 thousand, Hamamatsu (920 thousand) and Niigata (950 thousand) qualify as sustainable city populations.

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Table 4

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Figure 2

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5 Conclusions

While urban economists have carried out theoretical studies on optimal city size, empirical studies have not been well developed and have produced inconsistent results. In order for planners and policy makers to create a sustainable society, more empirical work needs to be done to test the desirability of a shift toward compact cities.

The main purpose of this study was to investigate empirically the optimal size of a city. Although we find an optimal city size attaining maximum total surplus (i.e. the difference between total benefits and total costs), our study differs from others in that we include social costs such as environmental pollution in total costs. Another important characteristic of our study is that the empirical models are obtained from theoretical models. In order to find the optimal city size, we used a data set of 262 Japanese metropolitan areas for the year 2000 and applied several regression models for total benefits and total costs. From our results, we pinpoint the following points as important.

First, from the regression analysis, coefficients of population (N) show the positive sign in both the total benefit function and the total cost function. Therefore, both total benefits and total costs increase as population increases. However, the degree of the coefficient between total benefit function and total cost function is different. As it is less than one in the total benefit function but more than one in the total cost function, at some degree of population, costs overtake benefits.

Second, the optimal city size, that attaining the maximum surplus, is 393,151 persons, much smaller than in previous studies. Our study is most distinguishable from others in that social costs are considered, which may account for our much smaller optimal city size. In fact, when we observe only social costs, the average social costs increase more sharply as population increases.

Last, the sustainable limit city size, in which total benefits equal total costs, is 1,057,412. This kind of city size closely reflects real metropolitan areas in Japan: Kumamoto, Shizuoka, Hamamatsu and Niigata, all of whose quality of life is considered relatively good.

In conclusion, a city's benefits and costs are so complex and there are so many factors affecting city size that it is difficult to pinpoint exactly what a city population should be. However, if we consider social costs, the ideal city has a much smaller population than many cities in the world today. Currently, there is a trend in urban planning based on the concept of compact cities. Our results might provide empirical evidence to show that this trend is a step in the right direction.

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Table 1 Unit Cost of Emission Type

Type of emission	CO_2	NO_X	SO_X	SPM
Unit	Yen/t-C	Thousand yen/t	Thousand yen/t	Thousand yen/t
Values	2,423	11,841	1,150	10,989
(Note): These values are obtained from original literature: Nakamura (1997) for CO_2, Small and Kazimi (1995) for NO_X, SO_X and SPM.				

Table 2 Statistics for the Analysis

Variable	Unit	Mean	Standard Deviation	Minimum	Maximum
Total Benefit (*TB*)	million yen	908,610	4,493,090	28,233	68,405,200
Total Cost (*TC*)	million yen	970,001	5,354,000	27,943	82,548,300
Household Expenditure (*HC*)	million yen	564,976	2,907,840	19,901	44,398,900
Population (*N*)	person	435,344	2,107,368	16,184	31,814,200
Price of exported output (*p*)	-	1.016	0.029	0.945	1.102
Housing price (*r*)	-	0.816	0.116	0.563	1.645
Tax rate (*t*)	index	58.661	19.348	17.337	137.459
Road density (TR_1)	km/km^2	0.100	0.038	0.024	0.228
Ratio of public transportation (TR_2)	-	0.113	0.088	0.014	0.641

Table 3 Regression Results: Coefficients and Standard Errors

	Model TB	Model TC	Model HC
	ln(TB/p)	*ln TC*	*ln HC*
Population (*ln N*)	0.985*** (0.008)	1.037*** (0.011)	1.001*** (0.008)
Road Density ($ln\ TR_1$)	0.067*** (0.020)	-	-
Ratio of Public Transportation ($ln\ TR_2$)	0.022 (0.014)	-	-
Price of Exported Output (*ln p*)	-	0.530*** (0.089)	0.836*** (0.064)
Housing Price (*ln r*)	-	0.470*** (0.089)	0.164*** (0.064)
Tax Rate (*ln t*)	-	0.015 (0.032)	0.027 (0.023)
Constant	7.992*** (0.137)	7.120*** (0.193)	7.014*** (0.134)
Adjusted R^2	0.987	0.980	0.989

(Note):
(1) Significant at 1% (***), 5% (**), 10% (*).

Table 4 Estimated Optimal City Size and Sustainable Limit City Size

Case	Lower Limit	Most Likely	Upper Limit
Optimal City Size	120,318	393,151	1,639,427
Sustainable Limit City Size	323,532	1,057,412	4,410,401

(Note):
(1) Optimal city population size is that at which a city attains maximum net benefits. Sustainable limit city population size is that at which a city's total benefits equal total costs.
(2) The upper and lower limits are obtained from the upper and lower bounds of the coefficient of a city's population (N) for both the total benefit (TB) function and the total cost function (TC). The upper limit is obtained by using the upper limit in the total benefit function and the lower limit in the total cost function. The lower limit is obtained by using the lower limit in total the benefit function and the upper limit in total cost function. The 20% confidence interval is used for both functions.

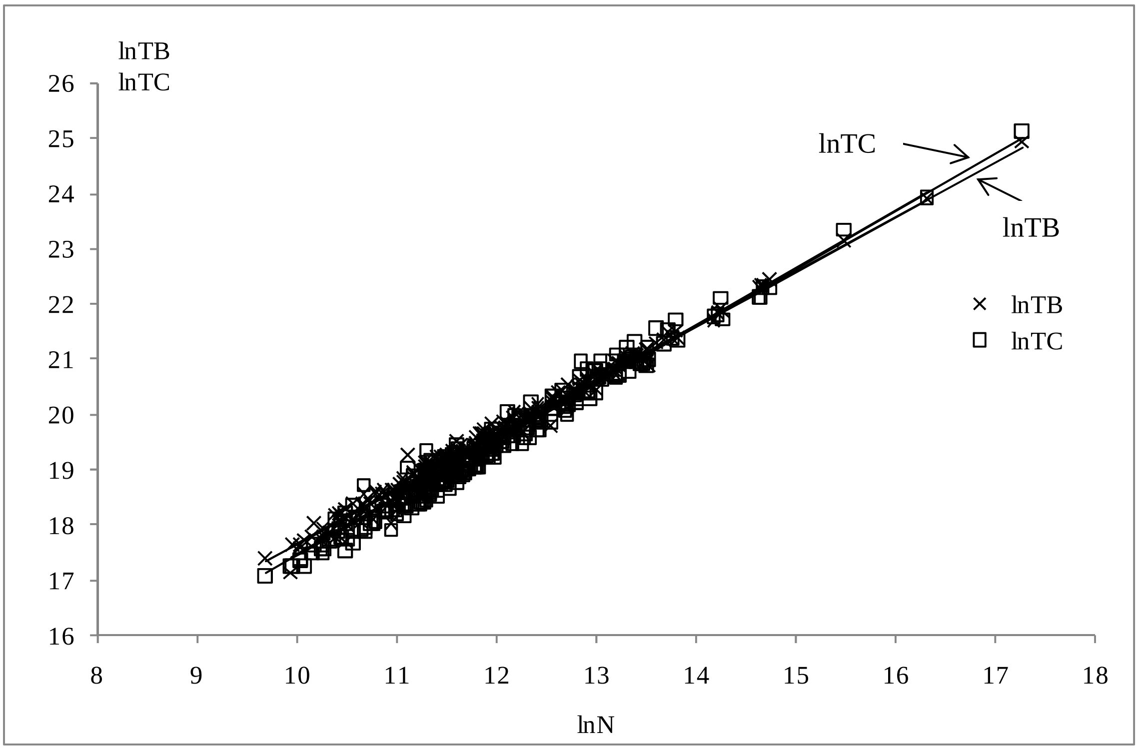

Figure 1 Distribution of the Actual Total Benefits and Total Costs

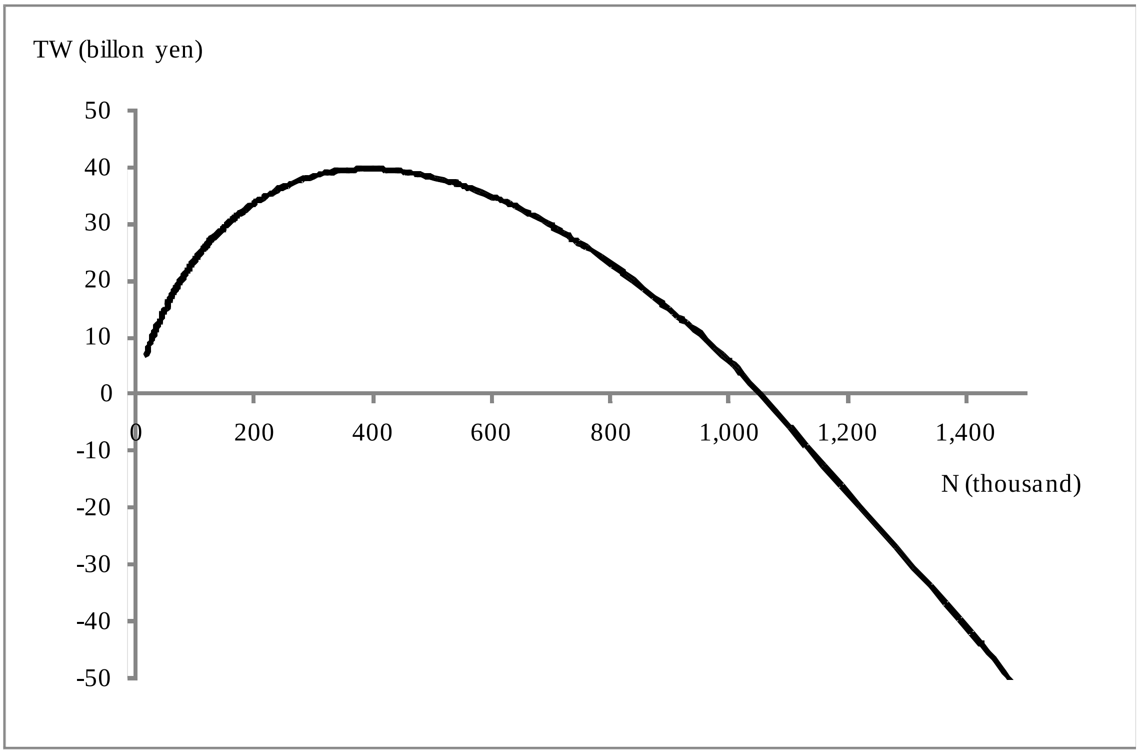

Figure 2 Estimated Total Surplus